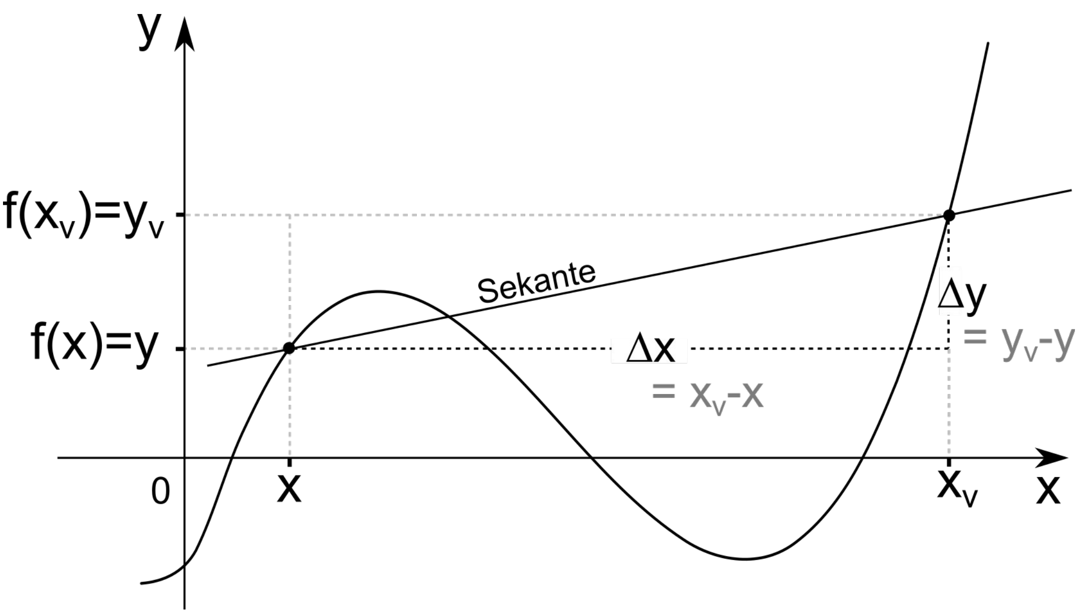
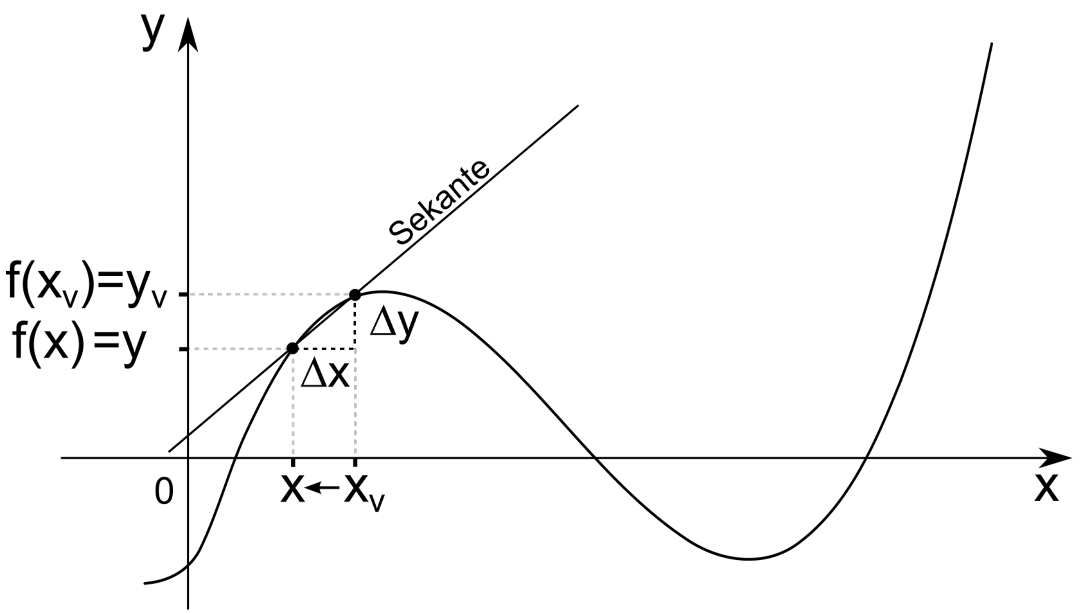
Differenzenquotient / Differentialquotient

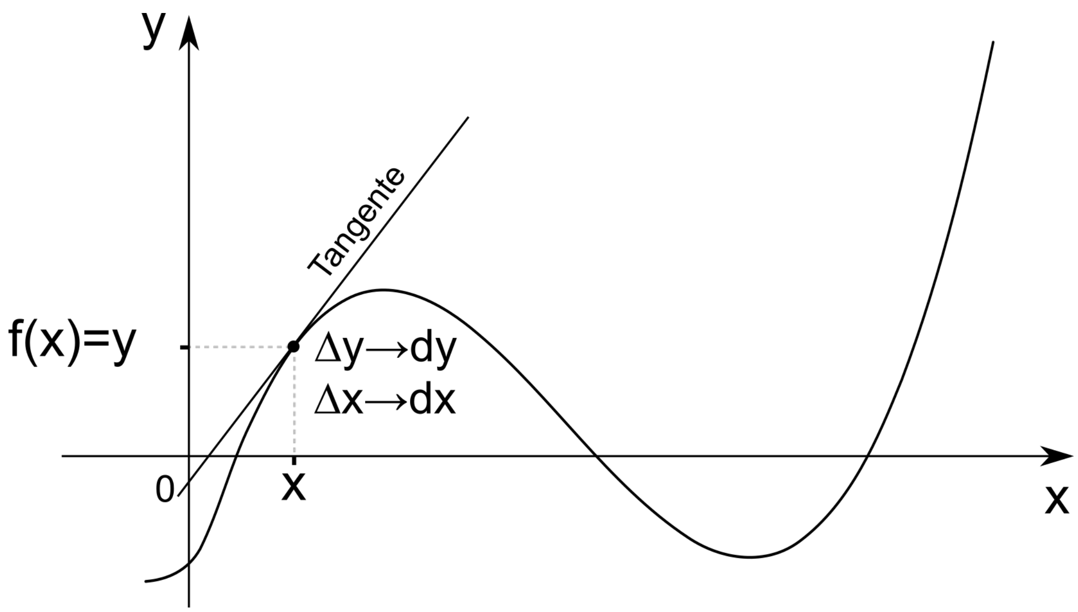
Durchschnittliche Steigung (Änderungsrate) zwischen x und x_v, also Steigung der Sekante durch die Punkte $P(x, f(x))$ und $P(x_v, f(x_v)) = (x_v, y_v) \Rightarrow m = \frac{\Delta y}{\Delta x} = \frac{y_v - y}{x_v - x} = \underbrace{\frac{f(x_v) - f(x)}{x_v - x}}_{Differenzenquotient}$





Momentane Steigung (Änderungsrate) an der Stelle x, also Steigung der Tangente an den Graphen im Punkt $P(x, f(x)) \Rightarrow m = \lim\limits_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \lim\limits_{x_v \to x}\left(\frac{y_v - y}{x_v - x}\right) = \underbrace{\lim\limits_{x_v \to x}\left(\frac{f(x_v) - f(x)}{x_v - x}\right)}_{\text{Differentialquotient}} =: f'(x)$

Tangente t an der Stelle x_0: $t : \mathbb{R} \to \mathbb{R},\ t(x) = f'(x_0)(x - x_0) + f(x_0)$

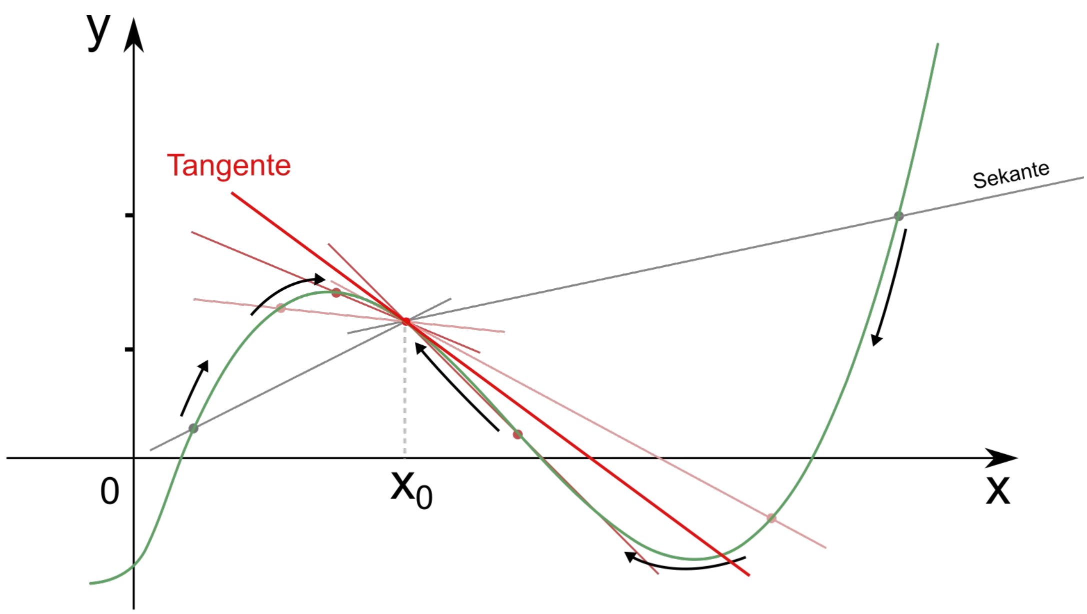


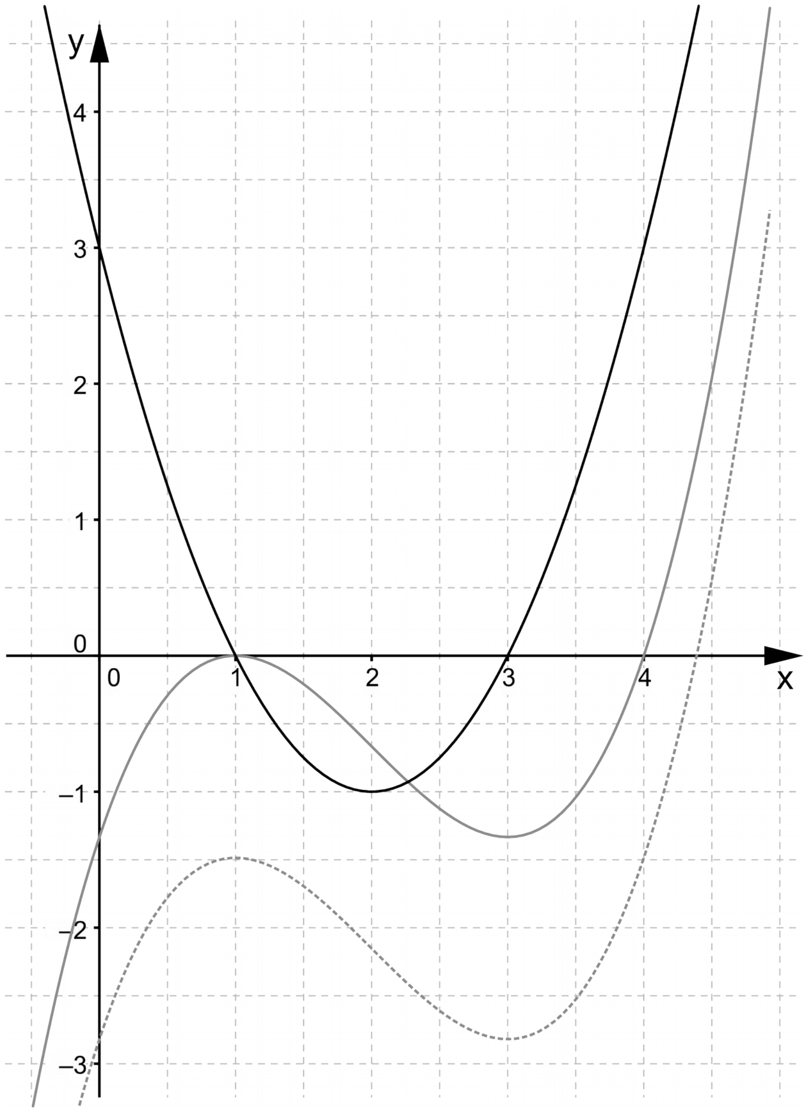
$$(a)' = \lim_{x_v \to x}\left(\frac{f(x_v) - f(x)}{x_v - x}\right) = \lim_{x_v \to x}\left(\frac{a - a}{x_v - x}\right) = 0$$

$$(x)' = \lim_{x_v \to x}\left(\frac{f(x_v) - f(x)}{x_v - x}\right) = \lim_{x_v \to x}\left(\frac{x_v - x}{x_v - x}\right) = 1$$

$$g(x) = af(x) \Rightarrow (a\,f(x))' = g'(x) = \lim_{x_v \to x}\left(\frac{g(x_v) - g(x)}{x_v - x}\right)$$

$$= \lim_{x_v \to x}\left(\frac{af(x_v) - af(x)}{x_v - x}\right) = \lim_{x_v \to x}\left(\frac{a(f(x_v) - f(x))}{x_v - x}\right) = a \lim_{x_v \to x}\left(\frac{(f(x_v) - f(x))}{x_v - x}\right) = a\,f'(x)$$


y
Tangente
Sekante
0
x_0
x


y
4
3
2
1
0
−1
−2
−3
0
1
2
3
4
x